Wheelset torsion mode

Assume a mode of vibration; left and right wheel:

$$\beta_{l} = \begin{pmatrix} whe_{k} \cdot z_{l} \\ 0 \\ -whe_{k} \cdot x_{l} \end{pmatrix} \qquad -\sqrt{(r^{2} - x_{l}^{2})} \leq z_{l} \leq \sqrt{(r^{2} - x_{l}^{2})} \\ -\sqrt{(r^{2} - z_{l}^{2})} \leq x_{l} \leq \sqrt{(r^{2} - z_{l}^{2})} \\ \beta_{r} = \begin{pmatrix} -whe_{k} \cdot z_{r} \\ 0 \\ whe_{k} \cdot x_{r} \end{pmatrix} \qquad -\sqrt{(r^{2} - x_{l}^{2})} \leq z_{r} \leq \sqrt{(r^{2} - x_{l}^{2})} \\ -\sqrt{(r^{2} - z_{l}^{2})} \leq x_{r} \leq \sqrt{(r^{2} - z_{l}^{2})} \\ \beta_{r} = \begin{pmatrix} -whe_{k} \cdot z_{r} \\ 0 \\ whe_{k} \cdot x_{r} \end{pmatrix} \qquad -\sqrt{(r^{2} - z_{l}^{2})} \leq x_{r} \leq \sqrt{(r^{2} - z_{l}^{2})} \\ -\sqrt{(r^{2} - z_{l}^{2})} \leq x_{r} \leq \sqrt{(r^{2} - z_{l}^{2})} \\ \beta_{r} = \begin{pmatrix} -whe_{k} \cdot z_{r} \\ 0 \\ whe_{k} \cdot x_{r} \end{pmatrix} \qquad -\sqrt{(r^{2} - z_{l}^{2})} \leq x_{r} \leq \sqrt{(r^{2} - z_{l}^{2})} \\ -\sqrt{(r^{2} - z_{l}^{2})} \leq x_{r} \leq \sqrt{(r^{2} - z_{l}^{2})} \\ \beta_{r} = \begin{pmatrix} -whe_{k} \cdot z_{r} \\ 0 \\ whe_{k} \cdot x_{r} \end{pmatrix} \qquad -\sqrt{(r^{2} - z_{l}^{2})} \leq x_{r} \leq \sqrt{(r^{2} - z_{l}^{2})} \\ -\sqrt{(r^{2} - z_{l}^{2})} \leq x_{r} \leq \sqrt{(r^{2} - z_{l}^{2})} \\ \beta_{r} = \begin{pmatrix} -whe_{k} \cdot z_{r} \\ 0 \\ whe_{k} \cdot x_{r} \end{pmatrix} \qquad -\sqrt{(r^{2} - z_{l}^{2})} \leq x_{r} \leq \sqrt{(r^{2} - z_{l}^{2})} \\ \beta_{r} = \begin{pmatrix} -whe_{k} \cdot z_{r} \\ 0 \\ whe_{k} \cdot x_{r} \end{pmatrix} \qquad -\sqrt{(r^{2} - z_{l}^{2})} \leq x_{r} \leq \sqrt{(r^{2} - z_{l}^{2})} \\ \beta_{r} = \begin{pmatrix} -whe_{k} \cdot z_{r} \\ 0 \\ whe_{k} \cdot x_{r} \end{pmatrix} \qquad -\sqrt{(r^{2} - z_{l}^{2})} \leq x_{r} \leq \sqrt{(r^{2} - z_{l}^{2})} \\ \beta_{r} = \begin{pmatrix} -whe_{k} \cdot z_{r} \\ 0 \\ whe_{k} \cdot x_{r} \end{pmatrix} \qquad -\sqrt{(r^{2} - z_{l}^{2})} \leq x_{r} \leq \sqrt{(r^{2} - z_{l}^{2})} \\ \beta_{r} = \begin{pmatrix} -whe_{k} \cdot z_{r} \\ 0 \\ whe_{k} \cdot x_{r} \end{pmatrix} \qquad -\sqrt{(r^{2} - z_{l}^{2})} \leq x_{r} \leq \sqrt{(r^{2} - z_{l}^{2})} \\ \beta_{r} = \begin{pmatrix} -whe_{k} \cdot z_{r} \\ 0 \\ whe_{k} \cdot x_{r} \end{pmatrix} \qquad -\sqrt{(r^{2} - z_{l}^{2})} \leq x_{r} \leq \sqrt{(r^{2} - z_{l}^{2})} \\ \beta_{r} = \begin{pmatrix} -whe_{k} \cdot z_{r} \\ 0 \\ whe_{k} \cdot z_{r} \end{pmatrix} \qquad -\sqrt{(r^{2} - z_{l}^{2})} \leq x_{r} \leq \sqrt{(r^{2} - z_{l}^{2})} \\ \beta_{r} = \begin{pmatrix} -whe_{k} \cdot z_{r} \\ 0 \\ whe_{k} \cdot z_{r} \end{pmatrix} \qquad -\sqrt{(r^{2} - z_{l}^{2})} \leq x_{r} \leq \sqrt{(r^{2} - z_{l}^{2})} \\ \beta_{r} = \begin{pmatrix} -whe_{k} \cdot z_{r} \\ 0 \\ whe_{k} \cdot z_{r} \end{pmatrix} \qquad -\sqrt{(r^{2} - z_{l}^{2})} \leq x_{r} \leq \sqrt{(r^{2} - z_{l}^{2})} \end{cases}$$

Where: whe_k is the pitch angle of the wheels.

Mass-orthonormalize one wheel: $\iint (\beta_l^T dM_l \beta_l) = \iint (\beta_l^T \rho b \beta_l) dx_l dz_l = J_x \cdot whe_k^2 = 1$

Where: $dM_l = \rho \cdot b \cdot dx_l \cdot dz_l$

 ρ = Density b = Wheel thickness

Mass-orthonormalize the whole wheelset gives:

$$J_x \cdot 2 \cdot whe_k^2 = 1 \implies whe_k = \frac{1}{\sqrt{2 \cdot J_x}}$$

The contact points of the wheels are located at:

$$\begin{aligned} x_l &= x_r = 0. \\ z_l &= z_r = r_o \end{aligned}$$

Which gives the mass-orthonormalized amplitude in contact point:

$$cpl_x = \frac{r_o}{\sqrt{(2 \cdot J_x)}}$$
 Left wheel
 $cpr_x = \frac{-r_o}{\sqrt{(2 \cdot J_x)}}$ Right wheel

Wheelset bending mode

Assume a mode of vibration; left and right wheel:

$$\beta_{l} = \begin{pmatrix} 0. \\ -whe_{f} \cdot z_{l} \\ 0. \end{pmatrix} \quad -r \leq z_{l} \leq r$$
$$\beta_{r} = \begin{pmatrix} 0. \\ whe_{f} \cdot z_{r} \\ 0. \end{pmatrix} \quad -r \leq z_{r} \leq r$$

Where: whe_f is the roll angle of the wheels.

Mass-orthonormalize the whole wheelset gives:

$$J_{\varphi} \cdot 2 \cdot whe_f^2 = 1 \implies whe_f = \frac{1}{\sqrt{(2 \cdot J_{\varphi})}}$$

The contact points of the wheels are located at:

 $\begin{aligned} x_l &= x_r = \overline{0}, \\ z_l &= z_r = r_o \end{aligned}$

Which gives the mass-orthonormalized amplitude in contact point:

$$cpl_y = \frac{r_o}{\sqrt{(2 \cdot J_{\varphi})}}$$
 Left wheel
 $cpr_y = \frac{-r_o}{\sqrt{(2 \cdot J_{\varphi})}}$ Right wheel

The axle-boxes are located at:

$$\begin{cases}
x_r = 0, \\
y_r = B_{kzba} - B_o \\
z_r = 0, \\
x_l = 0, \\
y_l = -y_r \\
z_l = 0.
\end{cases}$$

Which gives the mass-orthonormalized amplitude in the primary suspension:

$$kzba_{z} = \frac{B_{kzba} - B_{o}}{\sqrt{(2 \cdot J_{\phi})}}$$
Left wheel
$$kzba_{z} = \frac{B_{kzba} - B_{o}}{\sqrt{(2 \cdot J_{\phi})}}$$
Right wheel

Define two vibration modes:

```
mass m_flex_1 axl_111
    fq_torsion damp_torsion
    fq_bending damp_bending
```

Add coupling series flexibilities:

coupl m_flex_1 cp1_111l end_2 cpl_x 0. 0. 0. cply 0. coupl m_flex_1 cp1_111r end_2 $cpr_x 0. 0.$ 0 cpry 0. coupl m_flex_1 kzba_1111 end_2 0. 0. 0. 0. 0. 0. 0. 0. kzba_z 0. 0. 0. coupl m flex 1 kzba 111r end 2 0. 0. 0. 0. 0. 0. 0. 0. kzba_z 0. 0. 0. coupl m_flex_1 czba_1111 end_2 0. 0. 0. 0. 0. 0. 0. 0. kzba_z 0. 0. 0. coupl m_flex_1 czba_111r end_2 0. 0. 0. 0. 0. 0.

0. 0. kzba_z 0. 0. 0.